Analysis on Promotional Compaign Effects of Direct Bill Insert Advertising

Using a Transfer Function Time-Series Model

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Abstract

Promotional campaigns are often used by utilities industries to increase total sales level of their products or services. Evaluation of the effectiveness of the campaigns is a key component for the utilities to effectively use their resources because the campaigns normally require expenses. A transfer function for the time-series model was applied for an analysis of the direct bill insert advertising c to promote a new toilet replacement program for Dallas Water Utilities (DWU) customers. The analysis was based on the numbers of customers who participated in the program from August, 2007 to September, 2010. A point intervention function was used to indicate three times of the advertising campaigns, with the model to quantify the promotional effect. As a result, an exponential function was identified to describe the effect. The study showed that, after a promotional campaign, the participation rates were significantly increased, and then quickly shrunk with an exponential decreasing trend.

Introduction

Water Conservation has been an essential element of Dallas' long range water supply strategy since the early 1980's. According to the Energy Policy Act, all new residences constructed after 1992 are required to have water efficient toilets, faucets and showerheads. Water efficient toilets typically use 1.6 gallons per flush as opposed to older (pre-1992) toilets that can use anywhere from 3.5 to 7 gallons per flush -- saving more than 50% of water. US Census data indicated that less than 5% of single-family homes were constructed after 1995 in

Dallas (US Census, 2000, http://www.census.gov/main/www/cen2000.html). Over 244, 000 single-family homes were estimated to be built before 1992. In Dallas, single-family homes almost consume 40% of total water demand. Replacing old toilets with low-flow or high-efficiency models represents the significant reduction of water consumption. As far, there are approximately 160,000 single-family homes who are eligible to participate in the program. If converting all high-flow toilets to the low-flow models in the next decade, at least 16,000 single families should upgrade their toilets in every single year. In 2007, Dallas launched a toilet replacement program which offers up to $90 per toilet for single-family homes biult prior to 1992. Since it started, the program has been promoted on the DWU water conservation website; brochures were produced in different languages; and the City's staff provided television and radio interviews and appeared at community events to promote the program. Especially, a promotional campaign using direct bill insert significantly increased the participation rates over the last three years.

A better decision-making on retail business is important to assist the business development for retail industries. Market promotions are initiative to promote an increase in sales, usage of a product, or services. Also, the promotions cost money and are expected to gain the benefits from marketing. The effectiveness of promotional campaign with the direct bill inserts for the single-family toilet replacement program could be evaluated by a transfer function time-series model. This study tried to capture the complex relationships between the promotions and customer responses, and to answer the following questions: Was a past promotion effective? Will a proposed promotion be profitable? How will demand be affected by a planned promotion?

Data and Methodology

1. Data

The toilet replacement program began in August, 2007. The monthly number of customers who participated in the program from 2007 to 2010 was used for this analysis. During the period, as a major promotion measure, three direct bill inserts were mailed out to about 275,000 customers to promote the program in December, 2007, September, 2008, and December, 2009. Meanwhile, the website, television and radio interviews, and appearances at community events were also used to promote the program even though their promotion efforts were imitated. For simplicity, the study assumed that only bill insert advertising was the major promotion tool to significantly impact customer's participation.

2. Methodology

2.1 Time series models

The time series models are a combination of the autoregressive (AR) and moving-average (MA) processes. According to the model requirement for stationarity on a response series, i.e. a series being with a constant mean and variance over times, differencing or other transformations may be necessary. In these cases, the time series models are called the Auto-Regressive Integrated Moving Average (ARIMA) (Box and Jenkins, 1976). An ARIMA model can be used to forecast a value in a response series as a linear combination of its own past values, past errors. The models can also be applied to multivariate time series such that the models not only depend on the current and past values of a response series but also other explanatory series. The ARIMA models with one or more explanatory series are called transfer function models. How an explanatory series enters an ARIMA model is called its transfer function. The transfer function models can be described as below (SAS, OnlineDoc, 2002-2003) :

$$W_t = \mu + \sum \frac{\omega_i(B)}{\delta_i(B)} B^{k_i} X_{i,t} + \frac{\theta(B)}{\phi(B)} \alpha_i \qquad (1)$$

where, t indicates a time point; W_t is the response series Y_t or a difference of the response series; μ is a mean term; $X_{i,t}$ is the *i*th explanatory series or a difference of the *i*th explanatory series at time *t*; B is the backshift operator, i.e. $BX_t = X_{t-1}$; $\phi(B)$ is the autoregressive operator, represented as a polynomial in the backshift operator: $\phi(B) = 1 - \phi_1 B - ...... - \phi_p B^p$; $\theta(B)$ is the moving average operator, represented as a polynomial in the backshift operator: $\theta(B) = 1 - \theta_1 B - ...... - \theta_q B^q$, and α_t is a random error. $\omega_i(B)$ and $\delta_i(B)$ are component of the transfer function for the *i*th explanatory series.

The strategy for constructing a transfer function model is based on a three-step iterative cycle of model identification, estimation and diagnostic check proposed by Box and Jenkins (1976), and Haugh (1976). The transfer function models allow to building the relationship between the response series and the explanatory series. It includes identifying an adequate ARIMA model for both the response and explanatory series, identifying the systematic part of the transfer function model, introducing the lagged explanatory in the model, estimating the parameters and performing diagnostic checks. Finally, the goodness-of-fit statistics of AIC are estimated for model selection.

2.2 Intervention inputs

One special kind of the transfer function models is with one or multiple inputs as an indicator variable, usually coded with 1 or 0 to flag the occurrence of an event affecting the response series. It is also called an intervention model (Box and Tiao, 1975, and Hipel, *et. al.*, 1975). This model can be used to analyze an intervention impact. Normally, there are two commonly used types of intervention inputs (Leonard, 2001):

(1) Point intervention,

$$\begin{aligned} X_t &= 1, \quad if \quad t = time \\ X_t &= 0, \quad otherwise \end{aligned} \tag{2}$$

(2) Step intervention,

$$\begin{aligned} X_t &= 0, \quad if \quad t < time \\ X_t &= 1, \quad otherwise \end{aligned} \tag{3}$$

2.3 Transfer functions

Interventions are introduced in a time series model through a transfer function. The transfer function explains how the current and previous (lagged) values of intervention inputs cause deviations in an underlying time series process. A typical transfer function has a form as described in formula (1), $\frac{\omega_i(B)}{\delta_i(B)} B^{k_i}$. The overall influence of intervention inputs on the underlying time series is subsequently referred to as the intervention effect. The transfer function is a finite or infinite order function. $\omega_i(B)$ is the s*th* order *numerator* polynomial,

$\omega_i(B) = 1 - \omega_1 B - \omega_2 B^2 - ...... - \omega_s B^s$, and $\delta_i(B)$ is the r*th denominator* polynomial, $\delta_i(B)$ = $1 - \delta_1 B - \delta_2 B^2 - ...... - \delta_r B^r$. k_i is the pure time delay for the effect of the *i*th explanatory series; If r = 0, the transfer function is of finite order; otherwise, it is of infinite order. Commonly used denominator polynomial terms include one parameter exponential decay filters (Exp, *s = 0*, *r=1*) and two parameter filters (Wave, *s = 0*, *r = 2*).

Analysis and Results

1. Model identification

The identification is a primary step which is to determine whether or not a response series is stationary, what kind of lag structures of explanatory series should be entered in models, and their importance for forecasting. In this study, the number of participants for the toilet replacement program was examined for its stationarity by diagnosing autocorrelation, inverse, partial autocorrelation, and Chi-Square test (Shumway and Stoffer, 2000). The Chi-Square test for a white noise is not significant, with a p-value larger than Chi-Square equal to 0.2618 at lag 6. Autocorrelation, inverse, and partial autocorrelation also indicate the series is stationary, no differencing is needed. The intervention events of the direct bill inserts occurred in December, 2007, September, 2008 and December, 2009. An indicator variable for the promotion campaign was created to represent the intervention effect, which was coded as 1 for the months following the promotional months. Other months were coded as 0. The cross-correlations between the number of participants and the intervention input showed a significant relationship that means the promotion statistically impact customer responses for their participation.

2. Model fitting with transfer functions

For fitting a transfer function model, the numerator and denominator factors with a linear function of the occurrence and most recent impacts of promotion were used. Meanwhile, the different orders of the autoregressive (*p*) and moving average parameter (*q*) for the model residuals were fitted as well. Based on the selection criterion of AIC, the smaller value, the better model, the best transfer function model was fitted as shown in Table 1 and Figure 1.

The transfer function is described in the table to represent the promotion effect on the number of participants per month.

Below are SAS codes for modeling process:

```
proc arima data=toilet_02;
identify var=issu crosscorr=(prom season);
run;
estimate p=(0) q=(0) input=(/*season*/ (1)/(1)prom ) /*plot*/ method=ml;
run;
forecast lead=15 id=dates out=result_w interval=months;
run;
```

Table 1. Transfer Function Model Estimation

Parameter	Estimate	Standard Error	t-Value	Approx. Pr > |t|	Lag	Variable
MU	121.23	35.19	3.44	0.0006	0	Participation
NUM1	541.8	53.69	10.09	<.0001	0	Promotion
NUM1,1	290.78	67.71	4.29	<.0001	1	Promotion
DEN1,1	0.868	0.048	17.96	<.0001	1	Promotion

Figure1. SAS Output of Transfer Function Model

```
Estimated Intercept    121.2333

          Input Number 1
      Input Variable    promotion

         Numerator Factors
Factor 1:  541.803 - 290.776 B**(1)

        Denominator Factors
   Factor 1:   1 - 0.86766 B**(1)
```

3. Promotion effects

The numerator and denominator factors of the transfer function model can be expressed as 541.803-290.776 ×B, 1-0.86766 ×B, respectively. A Taylor series expansion formula was used to simplify the function to an exponential function. The processes are the following:

$$
\begin{aligned}
&\frac{\beta_0 - \beta_1 B}{1 - \alpha_1 B} \bullet X_t \\
&= \frac{541.8 - 290.8B}{1 - 0.87B} \bullet X_t \qquad (4) \\
&= (541.8 - 290.8B) \bullet (1 + 0.87B + 0.87^2 B^2 + 0.87^3 B^3 + \ldots + 0.87^n B^n) \bullet X_t \\
&= 541.8X_t + 180.6X_{t-1} + 0.87(180.6)X_{t-2} + 0.87^2(180.6)X_{t-3} + \ldots + 0.87^{n-1}(180.6)X_{t-n}
\end{aligned}
$$

Here, in the formula (4), X_t is an indicator of the promotion effect. The model indicates that, if no promotion of the direct bill inserts takes place, the average monthly participation rates are 121 (see Table 1). So when X_t is 1, the estimated promotional effect is 541.8, indicating more customer responses for the program in the following month after the promotion. The increased number was about 541 plus deviation from the model. When the next month X_{t-1} = 1, the estimated promotional effect is 180.6. When the third post-month X_{t-2} = 1, the estimated effect is 0.87(180.6), and so on so forth. Overall, the promotion is significant. For this case, the number of participation is increased by 541 in the first month after the promotion. Afterwards, the promotion effect shrinks with an exponential decreasing trend, as shown in Figure 2. If there is a promotion with a bill insert in early 2011, 3328 customers are expected to participate in the program over one year period, 1876 of them due to the advertising message. If the direct bill inserts advertising takes place twice in 2011, 4560 customers will take part in the program. The projection of the participation rates in the program with one and two-time promotions are shown in Table 2. It is assumed that the promotion occurs in December before a new year and June at the middle of a year.

The promotion of the direct bill inserts significantly impacts the customers who respond to the program. For the planning purposes, the promotion campaigns should be adjusted in accordance with the program budget availability or other considerations.

Figure 2. Projection from Transfer Function Model

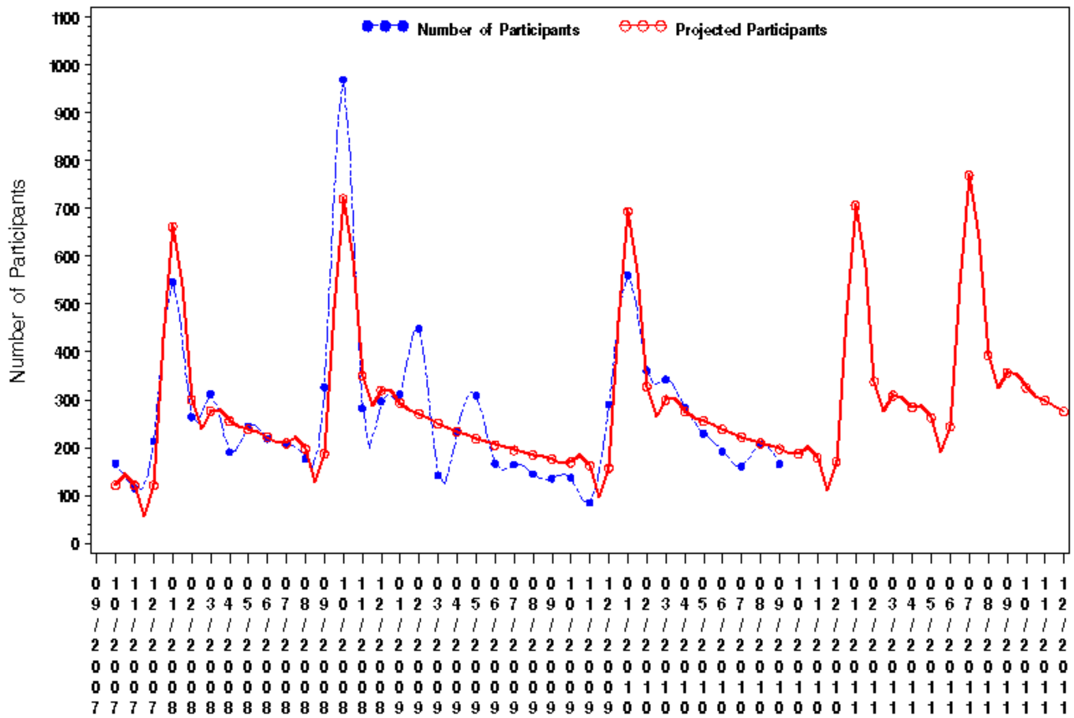

Table 2. Projection of Program Participation Rates with Promotion in 2011

Month	No promotion	One time promotion annually(1)	Two time promotion annually(2)
Jan	121	706	706
Feb	121	338	338
Mar	121	309	309
Apr	121	284	284
May	121	262	262
Jun	121	244	244
Jul	121	227	769
Aug	121	213	393
Sep	121	201	357
Oct	121	190	325
Nov	121	181	298
Dec	121	173	275
Total	1452	3328	4560

(1) the promotion takes place in December;
(2) the promotions take place in December and June.

Summary

In an effort to evaluate the promotional effectiveness of the direct bill insert advertising for the toilet replacement program in Dallas, a transfer function model was applied to the number of participation in the program from August, 2007 to September, 2010 to quantify the promotion effect.

The promotion of direct bill insert advertising significantly impacts customer participation into a water conservation program. If no direct bill insert advertising, the number of participating households is averaged 121 per month, and 1452 per year. With a promotion in 2011, 3328 customers are expected to participate in the program; when two promotions take place, 4560 homes are expected to take part in the program. Whenever the direct bill inserts takes place in a month, the customer participation in the following month is significantly increased; then the promotion effect quickly shrinks with an exponential decreasing trend.

This study can be used for a specific program planning for utilities. The promotion campaigns should be adjusted in accordance with the program plans and budget availability or other considerations.

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